# Package 'msBP'

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msBP-package	Bayesian nonparametric density estimation via Multiscale Bernstein Polynomials (msBP)

## **Description**

Performs Bayesian nonparametric multiscale density estimation and multiscale testing of group differences with multiscale Bernstein polynomials (msBP) mixtures as in Canale and Dunson (2016).

#### Author(s)

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#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

galaxy Galaxy velocities

#### **Description**

Dataset with the velocities of the 82 galaxies reported by Roeder (1990)

## Usage

data(galaxy)

## **Format**

A data frame with 82 observations and a single variable reporting the speed of galaxies (km/second)

## Source

Roeder, K. (1990) Density estimation with confidence sets exemplified by superclusters and voids in the galaxies, Journal of the American Statistical Association, 85: 617-624.

## References

Escobar, M.D. and West, M. (1995) Bayesian Density Estimation and Inference Using Mixtures. Journal of the American Statistical Association, 90: 577-588.

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#### **Examples**

data(galaxy)
str(galaxy)

msBP.compute.prob

Compute binary tree of probabilities

## **Description**

Compute the binary tree of probabilities using the multiscale stick-breaking process of Canale and Dunson (2016).

#### Usage

```
msBP.compute.prob(msBPtree, root = TRUE)
```

#### **Arguments**

msBPtree An object of the class msBPTree

root logical. if the root needs to be considered (default) or it should be cut (fixing

 $S_{01} = 0$ 

#### **Details**

Compute a binary tree of weights. The general weights for node h of scale s, is

$$\pi_{s,h} = S_{s,h} \prod_{r < s} (1 - S_{r,g_{shr}}) T_{shr}$$

where  $g_{shr} = \lceil h/2^{s-r} \rceil$  and  $T_{shr} = R_{r,g_{shr}}$  if  $(r+1,g_{shr+1})$  is the right daughter of node  $(r,g_{shr})$ , or  $T_{shr} = 1 - R_{r,g_{shr}}$  if  $(r+1,g_{shr+1})$  is the left daughter of  $(r,g_{shr})$ . An object of the msBPTree class is basically a list containing two objects of the class binaryTree: the S tree (representing the stoping probabilities) and the R tree (representing the proceed-right probabilities).

#### Value

An object of the class msbpTree.

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

#### See Also

msBP.rtree

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#### **Examples**

```
 S \leftarrow \text{structure(list(}\ T = \text{list(}1/8, \text{c(}1/3, 1/3), \ \text{c(}1/4, 1/4, 1/4, 1/4), \ \text{rep(}1,8)), \ \text{max.s=3), class} = \text{"binaryTree")} \\ R \leftarrow \text{structure(list(}\ T = \text{list(}1/2, \text{c(}1/2, 1/2), \ \text{c(}1/4, 1/2, 1/2, 1/2), \ \text{rep(}1,8)), \ \text{max.s=3), class} = \text{"binaryTree")} \\ RS \leftarrow \text{structure(list(}S = S, R = R), \ \text{class} = \text{"msbpTree")} \\ \text{probabilities} \leftarrow \text{msBP.compute.prob(}RS)
```

msBP.Gibbs

Gibbs sampling for density estimation for msBP model

## **Description**

Gibbs sampling for Markov Chain Motecarlo sampling from the posterior distribution of an msBP model.

## Usage

```
msBP.Gibbs(x, a, b, g0 = "normal", g0par=c(0,1), mcmc,
grid = list(n.points=40, low=0.001, upp=0.999), state=NULL, hyper,
printing=0, maxScale=5, ...)
```

## **Arguments**

X	the observed sample
a	scalar a parameter
b	scalar b parameter
g0	prior guess for the density of x. Currently only "normal", "unif", "gamma", and "empirical" are supported. From version 1.1 random paramters are also allowed (only with $g0="normal"$ ).
g0par	additional scalar parameters for g0. If "normal" corresponds to mean and standard deviation, if "uniform" to upper and lower bounds, if "gamma" to shape and rate parameters. If "empirical" this value is not used. From version 1.1 random parameters are also allowed (only with g0="normal").
mcmc	a list giving the MCMC parameters. It must include the following integers: nb giving the number of burn-in iterations, nrep giving the total number of iterations (including nb), and ndisplay giving the multiple of iterations to be displayed on screen while the C++ routine is running (a message will be printed every ndisplay iterations).
grid	a list giving the parameters for plotting the posterior mean density over a finite grid. It must include the following values: low and upp giving the lower and upper bound respectively of the grid and n.points, an integer giving the number of points of the grid
state	a list giving the current value of the parameters. This list is used if the current analysis is the continuation of a previous analysis or if we want to start the MCMC algorithm from some particular value of the parameters.

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hyper a list containing the values of the hyperparameters for a and b or for the param-

eters of the prior guess (only if g0="normal"). It must contains hyperprior, a list of three logical values determining if hyperpriors for a, b and g0 are used (TRUE) or if a, b, or g0 are fixed (FALSE), and hyperpar a list containing the hyperparameters for the hyperprior distributions: beta, gamma, delta, lambda, mu0, kappa0, alpha0, and beta0. See details. gridB is a grid of values for which the prior (and posterior) for b is evaluated with a Griddy Gibbs approach

(Ritter and Tanner, 1992). See details.

printing Vector of integers if the internal C++ function need to print what is doing

maxScale maximum scale of the binary trees.

... additional arguments.

#### **Details**

Before calling the proper C++ subrouting the function center the sample on an initial guess for the density of the data. If g0 = 'empirical' the data are transformed so that the expectation of the msBP prior is centered on the kernel density estimate of x.

The algorithm consists of two primary steps: (i) allocate each observation to a multiscale cluster, conditionally on the values of the weights (see also msBP.postCluster); (ii) update the weights, conditionally on the cluster allocations. All the procedure is written in C++ and additional R scripts are used to pre- and post-process the data and the output.

If hyper\$hyperpriors\$a or hyper\$hyperpriors\$b is true, additional hyperpriors for a and b are assumed. Specifically the algorithm implements  $a \sim Ga(\beta, \gamma)$  and  $b \sim Ga(\delta, \lambda)$ . For the former parameter the full conditional posterior distribution is available in closed form, i.e.

$$a|-\sim Ga\left(\beta+2^{s'+1}-1,\gamma-\sum_{s=0}^{s'}\sum_{h=1}^{2^s}\log(1-S_{s,h})\right),$$

while for the latter its full conditional posterior is proportional to

$$\frac{b^{\delta-1}}{B(b,b)^{2^{s+1}-1}} \exp \left\{ b \left( \sum_{s=0}^{s'} \sum_{h=1}^{2^s} \log \{R_{s,h}(1-R_{s,h})\} - \lambda \right) \right\},\,$$

where s' is the maximum occupied scale and B(p,q) is the Beta function. To sample from the latter distribution, a griddy Gibbs approach over the grid defined by hyper\$hyperpar\$gridB is used. See Ritter and Tanner (1992). From Version 1.1, if hyper\$hyperpriors\$base=TRUE and g0="normal" additional hyperpriors for the parameter of the centering normal density are assumed. Specifically the model is

$$y = \Phi(x;\mu,\sigma^2)$$
 
$$(\mu,\sigma^2) \sim N(\mu;\mu_0,\kappa_0\sigma^2) \text{I-Ga}(\sigma^2;\alpha_0,\beta_0)$$

and an additional step simulating the values of  $\mu$  and  $\sigma^2$  from their conditional posterior distribution is added to the Gibbs sampler of Canale and Dunson (2016). Specifically, a Metropolis-Hastings step with proposal equal to the prior is implemented.

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#### Value

A list containing

density A list containing postMeanDens, the posterior mean density estimate evaluated

over xDens and postLowDens and postUppDens, the lower and upper pointwise

95% credible bands.

mcmc A list containing the MCMC chains: dens is a matrix (nrep-nb) times n. grid, a

and b are the vectors with the MCMC chains for the two parameters (if hyperprior was TRUE), scale is a matix where each column is a MCMC chain of the total mass for each scale, R and S, are matrices where each column in the tree2vec form of the corresponding trees, weights is a matrix where each column is the tree2vec form of the corresponding tree of weights, s and h are matrices where

each column is the MCMC chain for the node labels for a subject.

postmean A list containing posterior means over the MCMC samples of a, b, and of all

binary trees

fit A list containing the LPML, mean and median of the log CPO.

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

Ritter C., Tanner M. (1992). "Facilitating the Gibbs Sampler: the Gibbs Stopper and the Griddy-Gibbs Sampler." Journal of the American Statistical Association, 87, 861-868.

#### See Also

```
msBP.postCluster
```

#### **Examples**

```
## Not run:
data(galaxy)
galaxy <- data.frame(galaxy)</pre>
speeds <- galaxy$speed/1000
set.seed(1)
#with fixed g0 and random a, b
fit.msbp.1 <- msBP.Gibbs(speeds, a = 10, b = 5, g0 = "empirical",
mcmc=list(nrep = 10000, nb = 5000, ndisplay = 1000),
hyper=list(hyperprior=list(a = TRUE, b = TRUE, g0 = FALSE),
hyperpar=list(beta=5,gamma = 1,delta = 1,lambda = 1)),
printing = 0, maxS = 7, grid = list(n.points = 150, low = 5, upp = 38))
#with random a, b and hyperparameters of g0
fit.msbp.2 <- msBP.Gibbs(speeds, a = 10, b=5, g0 = "normal",
mcmc=list(nrep = 10000, nb = 5000, ndisplay = 1000),
hyper=list(hyperprior = list(a = TRUE, b = TRUE, g0 = TRUE),
 hyperpar = list(beta = 50, gamma = 5, delta = 10, lambda = 1,
```

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```
gridB = seq(0, 20, length = 30),
mu0 = 21, kappa0 = 0.1, alpha0 = 1, beta0 = 20)),
printing = 0, maxS = 7, grid = list(n.points = 150, lo w= 5, upp = 38))
hist(speeds, prob=TRUE,br=10, ylim=c(0,0.23), main="", col='grey')
points(fit.msbp.1$density$postMeanDens~fit.msbp.1$density$xDens, ty='1', lwd=2)
points(fit.msbp.1$density$postLowDens~fit.msbp.1$density$xDens, ty='1',lty=2, lwd=2)
points(fit.msbp.1$density$postLowDens~fit.msbp.1$density$xDens, ty='1',lty=2, lwd=2)
hist(speeds, prob=TRUE,br=10, ylim=c(0,0.23), main="", col='grey')
points(fit.msbp.2$density$postMeanDens~fit.msbp.2$density$xDens, ty='1', lwd=2)
points(fit.msbp.2$density$postUppDens~fit.msbp.2$density$xDens, ty='1',lty=2, lwd=2)
points(fit.msbp.2$density$postLowDens~fit.msbp.2$density$xDens, ty='1',lty=2, lwd=2)
## End(Not run)
```

msBP.nrvTrees

Nesting of the sample through the tree

## **Description**

Compute the path of each subject in the binary tree of weights and returns 3 tree: the n tree, the r tree, and the v tree (see values

#### Usage

```
msBP.nrvTrees(sh, maxS = max(sh[,1]))
```

#### **Arguments**

sh A matrix with 2 columns and a number of rows equal to the sample size denoting

the scale and node labels of each unit

maxS Upper bound for the scale

#### Value

A list containing tree objects of the class binaryTree. n is the tree containing at each node the number of subjects allocated to that node, r is the tree containing at each node the number of subjects that went right at that node, and v is the tree containing at each node the number of subjects that passed through that node.

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

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#### **Examples**

```
sh <- cbind(c(2,2,2,3,3,3,3,3,3), c(1,2,2,1,2,3,4,5,6,7)) nrv.trees <- msBP.nrvTrees(sh) plot(nrv.trees$n)
```

msBP.pdf Random probability density function and random cumulative distribution function from a msBP

## **Description**

Compute the density and the cumulative distribution functions of a random density drawn from an msBP(a,b) process

## Usage

```
msBP.pdf(weights, n.points, y)
msBP.cdf(weights, n.points, log, y)
```

## **Arguments**

weights	An object of the class binaryTree containing probability weights
n.points	Length of the grid over (0,1) in which calculate the value of the random density
log	Logical. TRUE for computing the log-cdf
У	Vector of values in which the random density is evaluated. If used, n.points is not considered.

#### Value

Vector of size n.points or length(y)

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

#### See Also

```
msBP.rtree
```

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#### **Examples**

```
prob <-structure(list( T = list(0.15, c(0.05,0.05), c(0.05,0.2,0.1,0.1),
c(0,0,0.3,0,0,0,0) ), max.s=3), class = "binaryTree")
density <- msBP.pdf(prob, 100)
probability <- msBP.cdf(prob, 100)
par(mfrow=c(1,2))
plot(density$dens~density$y, ty='l', main = "pdf")
plot(probability$prob~density$y, ty='l', main = "cdf")</pre>
```

msBP.postCluster

Posterior cluster allocation

## **Description**

Perform the posterior multiscale cluster allocation conditionally on a tree of weights. See Algorithm 1 in Canale and Dunson (2016).

#### Usage

```
msBP.postCluster(y, weights)
```

#### **Arguments**

y the sample of individials to be allocated to binary tree structure
weights the binary tree of weights (summing to one). An object of the class msBPTree

## Details

conditionally on the weights contained in weights, each subject in y is allocated to a multiscale cluster using Algorithm 1 of Canale and Dunson (2016). It relies on a multiscale modification of the slice sampler of Kalli et al. (2011).

#### Value

a matrix with length(y) row and two columns, denoting the scale and node within the scale, respectively.

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

Kalli, M., Griffin, J., and Walker, S. (2011), "Slice sampling mixture models," Statistics and Computing, 21, 93-105.

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#### See Also

```
msBP.Gibbs
```

## **Examples**

```
set.seed(1)
y <- rbeta(30, 5, 1)
weights <-structure(list(
T = list(0, c(0,0.10), c(0.0,0.3,0.6)), max.s=2),
class = 'binaryTree')
sh <- msBP.postCluster(y, weights)
clus.size <- msBP.nrvTrees(sh)$n
plot(clus.size)</pre>
```

msBP.rsample

Random numbers from a random msBP densty

## **Description**

Random numbers generation from a random density drawn from a msBP process.

## Usage

```
msBP.rsample(n, msBPtree)
```

## **Arguments**

n Size of the sample to be generated msBPtree An object of the class msBPtree

## Value

A vector containing the random sample

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

#### See Also

```
msBP.rtree
```

msBP.rtree

#### **Examples**

```
rand.tree <- msBP.rtree(50,2, 4)
rand.samp <- msBP.rsample(50, rand.tree)
hist(rand.samp, prob=TRUE)
prob <- msBP.compute.prob(rand.tree)
density <- msBP.pdf(prob, 100)
points(density$dens~density$y, ty='l', col=4)</pre>
```

msBP.rtree

Random msBP tree

## Description

Draw a random tree from the msBP process

#### Usage

```
msBP.rtree(a, b, max.s = 10)
```

## Arguments

a Scalar parameter
b Scalar parameter

max.s Maximum depth of the random trees

#### Value

An object of the class msBPTree

#### References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

## See Also

```
msBP.rsample, msBP.compute.prob
```

## **Examples**

```
msBP.rtree(2, 2, 4)
```

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msBP.test Multiscale testing of group differences	
---	--

## **Description**

Performs multiscale hypothesis testing of difference in the distribution of two groups using msBP prior.

## Usage

```
msBP.test(y, a, b, group, priorH0 = 0.5,
mcmc, maxScale = 5, plot.it = FALSE, ...)
```

#### **Arguments**

У	The pooled sample of observations
a,b	Parameters of the msBP prior
group	Vector of size length(y) with 0 and 1 denoting the group membership.
priorH0	Prior gues for the probability of H0
mcmc	a list giving the MCMC parameters. It must include the following integers: nb giving the number of burn-in iterations, nrep giving the total number of iterations (including nb)., and ndisplay giving the multiple of iterations to be displayed on screen while the MCMC is running (a message will be printed every ndisplay iterations).
maxScale	maximum scale of the binary trees.
plot.it	logical. If TRUE a plot of the posterior mean probability of H0 is produced
	additional arguments to be passed.

## Value

a list containing

Ps a matrix with maxScale rows and mcmc\$nrep columns with the MCMC draws

of posterior probabilities of H0 for each scale

Ps the posterior mean probabilities of H0 for each scale.

## References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

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#### **Examples**

```
set.seed(1)
y <- runif(100)
g <- c(rep(0,50), rep(1,50))
mcmc <- list(nrep = 5000, nb = 1000, ndisplay = 500)
## Not run:
test.res <- msBP.test(y, 5, 1, g, mcmc=mcmc, plot.it = TRUE)
## End(Not run)</pre>
```

msBP.tree

Creating an msBPTree

#### **Description**

Create an object of the class msBPTree

#### Usage

```
msBP.tree(max.s = 10)
```

## **Arguments**

max.s

Maximum depth of the binary tree

#### **Details**

An object of the class msbpTree is a list of 5 elements that represent a draw from a msBP(a,b) prior as introduced by Canale and Dunson (2016). The first two elements are the trees of the stopping and descending-to-the-right probabilities, respectively. Both are object of the class binaryTree. The third and fourth argument are the hyperparameters of the msBP prior, namely a and b. The last value is an integer with the maximum depth of both the trees.

#### Value

An object of the class msBPTree with zero at all nodes and a=b=NULL.

## References

Canale, A. and Dunson, D. B. (2016), "Multiscale Bernstein polynomials for densities", Statistica Sinica, 26(3), 1175-1195.

Canale, A. (2017), "msBP: An R Package to Perform Bayesian Nonparametric Inference Using Multiscale Bernstein Polynomials Mixtures". Journal of Statistical Software, 78(6), 1-19.

#### See Also

```
msBP.rtree
```

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## **Examples**

```
tree <- msBP.tree(2)</pre>
```

tree2vec

Conversions between tree and vector

## **Description**

Convert a binary tree object into a vector and \_vice versa\_

## Usage

```
tree2vec(tree)
vec2tree(vec)
```

## **Arguments**

tree An object of the class binaryTree

vec A vector of numbers. It must have size 2<sup>s</sup> - 1, with s an integer.

## **Details**

An object of the class binaryTree is a binary tree containing at each node a value.

## Value

A vector of size  $2^{D+1} - 1$ , where D is the depth of the binary tree, or a binary tree with depth  $\log_2(\text{length(vec)} + 1)$ .

## **Examples**

```
tree <- vec2tree(1:(2^5 - 1))
vector <- tree2vec(tree)</pre>
```

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